Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/22**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 7 5 6 7 2 A 0 1 1 6

Pearson

1. (i) The table below is a Cayley table for the group G with operation $\circ$

$\circ$	a	b	c	d	e	f
a	d	c	b	a	f	e
b	e	f	a	b	c	d
c	f	e	d	c	b	a
d	a	b	c	d	e	f
e	b	a	f	e	d	c
f	c	d	e	f	a	b

(a) State which element is the identity of the group.

(1)

(b) Determine the inverse of the element $(b \circ c)$

(2)

(c) Give a reason why the set $\{a, b, e, f\}$ cannot be a subgroup of G. You must justify your answer.

(1)

(d) Show that the set $\{b, d, f\}$ is a subgroup of G.

(2)

(ii) Given that H is a group with an element x of order 3 and an element y of order 6 satisfying

$$yx = xy^5$$

show that $y^3xy^3x^2$ is the identity element.

(3)

(1)(i)(a) d ① ← column / row d are unchanged

(b) $(b \circ c)^{-1} = a^{-1}$ ①

$a^{-1} = a$ ① ← $a \circ a = d \therefore a^{-1} = a$

(c) The set cannot be a subgroup because a subgroup must contain the identity element of the group, and d is not in $\{a, b, e, f\}$ ①

Question 1 continued

(d)

$\circ$	b	d	f
b	f	b	d
d	b	d	f
f	d	f	b

① $\{b, d, f\}$ is closed under $\circ$ because the table contains no elements outside of the set.

d is the identity element.

b and f are inverse because $b \circ f = d$.

① for all 3 statements

(ii) $y^3xy^3x^2 = y^2yxy^3x^2$

$= y^2xy^5y^3x^2$ ① $yx = xy^5$

$= yyxy^8x^2$

$= yxy^5y^8x^2$ $yx = xy^5$

$= xy^5y^5y^8x^2$ ①

$= xy^{18}x^2$

$= xex^2$

$= x^3$

$= e$ ①

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

2. Tiles are sold in boxes with 21 tiles in each box.

The tiles are laid out in x rows of 5 tiles and y rows of 6 tiles.

All the tiles from a box are used before the next box is opened.

When all the rows of tiles have been laid, there are n tiles left in the last opened box.

(a) Write down a congruence expression for n in the form

$$ax + by \pmod{c}$$

where a, b and c are integers.

(1)

Given that

- exactly 43 rows of tiles are laid
- there are no tiles left in the last opened box

(b) use your congruence expression to determine the minimum number of rows of 6 tiles laid.

(5)

(a) $n \equiv 5x + 6y \pmod{21}$ 1 ← as the number left in the box must be < 21

(b) $x + y = 43$ 1 ← total rows laid

$n \equiv 5(x+y) + y \pmod{21}$

$\equiv 5 \times 43 + y \pmod{21}$ 1

43 mod 21 = 1 (21 × 2 = 42 r 1)

$\equiv 5 \times 1 + y \pmod{21}$

$\equiv 5 + y \pmod{21}$ 1

$n \equiv 0 \pmod{21}$ because there are no tiles left over

$0 \equiv 5 + y \pmod{21}$

$y \equiv -5 \pmod{21}$ 1 so minimum value for y is 16 1

$-5 \bmod 21 = 21 - 5$

Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$\mathbf{A} = \begin{pmatrix} 3 & k \\ -5 & 2 \end{pmatrix}$$

where k is a constant.

Given that there exists a matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix where

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{pmatrix} 8 & 0 \\ 0 & -3 \end{pmatrix}$$

(a) show that $k = -6$

(3)

(b) determine a suitable matrix $\mathbf{P}$

(4)

(a) Using $P^{-1}AP$, eigenvalues are 8 and -3 ①

$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & k \\ -5 & 2-\lambda \end{vmatrix} = 0$

$0 = (3-\lambda)(2-\lambda) - (k \times -5)$

$0 = \lambda^2 - 5\lambda + 6 + 5k$ ①

$6 + 5k = -8 \times 3$ ← real part $(6+5k)$ is equal to $a \times b$ where $(\lambda + a)(\lambda + b) = 0$.

$5k = -30$

$k = -6$ ①

P 7 5 6 7 2 A 0 8 1 6

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Question 3 continued

(b) When $\lambda = -3$: $Av = \lambda v$

$$\begin{pmatrix} 3 & -6 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -3\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} 3x - 6y &= -3x \\ -5x + 2y &= -3y \end{aligned}\right\} \quad x = y \quad \text{so} \quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

1 1

When $\lambda = 8$: $Av = \lambda v$

$$\begin{pmatrix} 3 & -6 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 8\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} 3x - 6y &= 8x \\ -5x + 2y &= 8y \end{aligned}\right\} \quad 5x = -6y \quad \text{so} \quad v = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

1

$\therefore P = \begin{pmatrix} 6 & 1 \\ -5 & 1 \end{pmatrix}$ 1

(Total for Question 3 is 7 marks)

4. A circle C in the complex plane has equation

$$\left|z-(-3+3\mathrm{i})\right| = \alpha\left|z-(1+3\mathrm{i})\right|$$

where α is a real constant with $\alpha > 1$

Given that the imaginary axis is a tangent to C

(a) sketch, on an Argand diagram, the circle C

(2)

(b) explain why the value of α is 3

(1)

The circle C is contained in the region

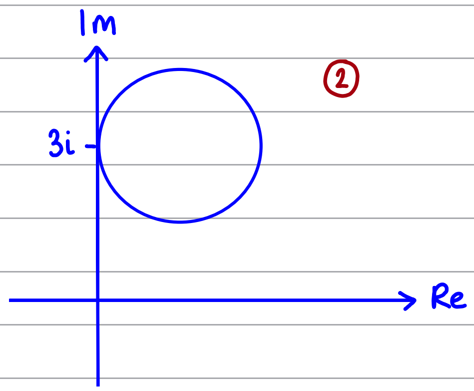
$$R = \left\{z \in \mathbb{C} : \beta \leqslant \arg z \leqslant \frac{\pi}{2}\right\}$$

(c) Determine the maximum value of β

Give your answer in radians to 3 significant figures.

(6)

(a)



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(b) Circle must touch imaginary axis at 3i

$|3i - (-3+3i)| = \alpha|3i - (1+3i)|$

$|3| = \alpha|-1|$

$3 = \alpha$ 1 $\leftarrow \alpha > 1$, so -3 is not valid

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Question 4 continued

(c) $z = x + yi$

$$|x + yi - (-3 + 3i)| = 3|x + yi - (1 + 3i)|$$

$$|(x+3) + (y-3)i| = 3|(x-1) + (y-3)|$$

$$(x+3)^2 + (y-3)^2 = 9(x-1)^2 + 9(y-3)^2$$

$$x^2 + 6x + 9 + y^2 - 6y + 9 = 9x^2 - 18x + 9 + 9y^2 - 54y + 81$$

$$0 = 8x^2 - 24x + 8y^2 - 48y + 72$$

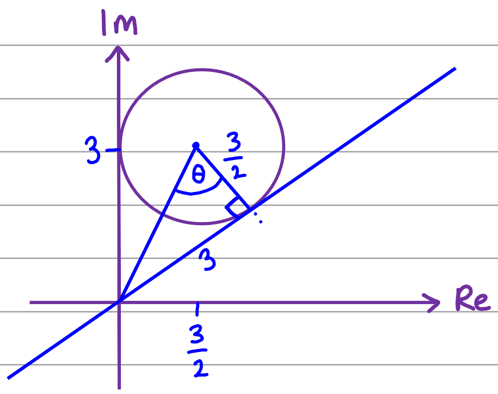
$$0 = x^2 - 3x + y^2 - 6y + 9$$

$$0 = (x - \tfrac{3}{2})^2 - \tfrac{9}{4} + (y-3)^2 - 9 + 9$$

$$\tfrac{9}{4} = (x - \tfrac{3}{2})^2 + (y-3)^2 \quad ①$$

$\therefore$ Centre is $\left(\frac{3}{2}, 3\right)$ ①

$\therefore$ Radius is $\sqrt{\frac{9}{4}} = \frac{3}{2}$ ①



$\beta = \theta - \phi$

$\theta = \tan^{-1}\left(\frac{3}{3/2}\right) = \tan^{-1}(2)$ ①

$\phi = \sin^{-1}\left(\frac{3/2}{\sqrt{(3/2)^2 + 3^2}}\right) = \sin^{-1}\left(\frac{\sqrt{5}}{5}\right)$

length of hypotenuse

$\beta = \theta - \phi$

$\beta = \tan^{-1}(2) - \sin^{-1}\left(\frac{\sqrt{5}}{5}\right)$ ①

$\beta = 0.644$ ①

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

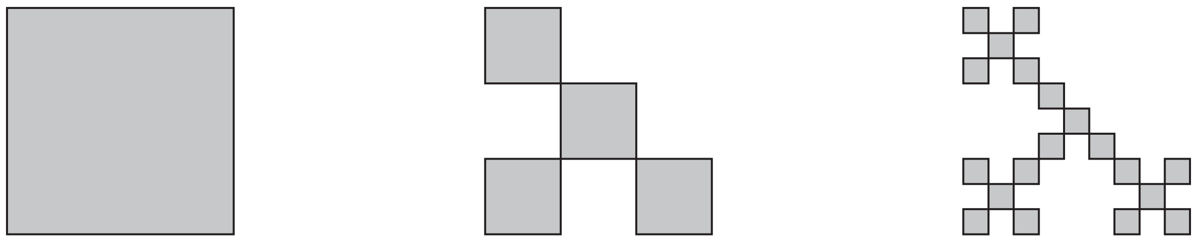


Figure 1

Figure 1 shows the first three stages of a pattern that is created by a recursive process.

The process starts with a square and proceeds as follows

- each square is replaced by 5 smaller squares each $\frac{1}{9}$th the size of the square being replaced
- the 5 smaller squares are the ones in each corner and the one in the centre
- once each of the squares has been replaced, the square immediately to the right and above the centre square of the pattern is then removed

Let u_n be the number of squares in the pattern in stage n, where stage 1 is the original square.

(a) Explain why u_n satisfies the recurrence system

$$u_1 = 1 \qquad u_{n+1} = 5u_n - 1 \qquad (n = 1, 2, 3, \ldots)$$

(2)

(b) Solve this recurrence system.

(5)

Given that the initial square has area 25

(c) determine the total area of all the squares in stage 8 of the pattern, giving your answer to 2 significant figures.

(2)

(a) In the first stage there is 1 square so $u_1 = 1$.

Each square from u_n to u_{n+1} is replaced by 5 smaller squares, so $u_{n+1} = 5u_n$. ① for 2/3 points

But one square is removed, so $u_{n+1} = 5u_n - 1$. ① for 3/3 points

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Question 5 continued

(b) $u_{n+1} = 5u_n - 1$

$u_{n+1} = CF + PS$

AE : $\lambda - 5 = 0 \Rightarrow \lambda = 5$ ①

General solution to $u_{n+1} = au_n$ is $u_{n+1} = ca^n$.

$CF = c \times \lambda^n = c \times 5^n$ ①

Try $PS = k$:

$k = 5(k) - 1$

$k = \frac{1}{4}$ ①

$u_n = CF + PS = c \times 5^n + \frac{1}{4}$

When $n = 1$, $u_n = 1$.

$1 = c \times 5^1 + \frac{1}{4}$

$\frac{3}{4} = 5c$

$\frac{3}{20} = c$ ①

$\therefore u_n = \frac{3}{20} \times 5^n + \frac{1}{4}$ ①

Turn over

P 7 5 6 7 2 A 0 1 5 1 6

Question 5 continued

(c) Each square in n has area $\frac{25}{9^{n-1}}$ so total area:

$$\frac{25}{9^{n-1}} \times u_8 = \frac{25}{9^7} \times \left(\frac{3}{4} \times 5^7 + \frac{1}{4}\right) \quad 1$$

$$= 0.31 \quad 1$$

(Total for Question 5 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS